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Quality Factor, Bandwidth, and Harmonic Attenuation of Pi Networks

The author looks at several definitions of Q, and makes some interesting discoveries about what these calculations can tell us about the design of pi networks..

I recently became interested in pi networks, a type of impedance-matching resonant network that provides harmonic attenuation and is often used in tube-type Amateur Radio transmitters. I was particularly interested in the relationship between the quality factor, Q, and the bandwidth of these networks. In various editions of *The ARRL Handbook for Radio Communications*, I found three definitions of Q, all different. None of the three were very good predictors of the bandwidths of pi networks for most values of the load and source impedances. During these explorations, however, I discovered that a modification of one of the three provided a much better predictor of bandwidth. Subsequently, I was able to derive this modified form of Q theoretically.

While the bandwidth of pi networks is interesting, a perhaps more important characteristic of these networks is their ability to attenuate harmonics present in the signals passing through them. This paper presents data on the harmonic attenuation of these networks as a function of their quality factors.

Finally, I investigated whether the methods used in this paper could be extended to more complex networks. For pi-L networks, they yield a value for Q that fairly accurately predicts bandwidth but not harmonic attenuation. They also yield quality factors for more complicated networks, but these values do not seem to have much relationship with bandwidth.

Quality Factor of Impedance-Matching Networks

Impedance-matching networks are characterized by, among other things, their design frequency (the frequency at which the input and output impedances are matched) and by the quality factor, Q. Quality factor is defined in two different ways. The first, and perhaps most common way, to define Q is given by Equation 1.

$$Q = 2\pi \times f_0 \times \frac{\text{Average energy stored in reactive elements}}{\text{Power dissapted by lossy elements}} \quad \text{[Eq 1]}$$

where f_0 is the design frequency (in which the output and input impedances are matched), and dissipated power is the electrical energy dissipated per second, that is, converted into some other form of energy, such as thermal or radiation. (Dissipated power divided by frequency is the energy dissipated per cycle.)

Equation 2 gives a second definition of Q.

$$Q = \frac{f_0}{BW} \quad \text{[Eq 2]}$$

where BW is the 3 dB bandwidth, which is the width between the upper and lower frequencies at which the response of the circuit is down 3 dB from its response at f_0.

Circuit-analysis textbooks show that these two definitions are exactly equivalent for simple series and parallel *RLC* circuits. For circuits using more than two reactive components, such as the pi network, it is unclear (to me at least) that the definitions given by Equations 1 and 2 are even approximately equivalent. This is important because the Q of a more-complicated matching network is most easily calculated using Equation 1, because this calculation can only be performed at one frequency, whereas the bandwidth and the network's ability to attenuate harmonics of the design frequency are usually the more important parameters in the design process. That is, it seems to me that Q is useful for practical design work in so much as it provides information about the bandwidth and/or harmonic attenuation of a circuit.

Wes Hayward, W7ZOI, in his ARRL book *Introduction to Radio Frequency Design* has the following discussion of Q.[1]

> "Q is not well defined for networks with three or more reactive components. Still, Q is a frequently used parameter in the design equations for more complex networks. The meaning of Q is different when applied to such networks. It is the ratio of a

[1]Notes appear on page 35

resistance to a reactance when looking into one end of the network at one frequency. Network reduction methods are always used. The user should not deduce the bandwidth of the network by the Q used for design."

After puzzling about this for some time, I decided to look into the relationship between Q and bandwidth for a specific matching circuit, the pi network (which includes three reactive components).

Quality Factor and Bandwidth of Pi Networks

Figure 1 shows a pi-network circuit configuration often used to match a load impedance, R_L, to a source impedance, R_S. A common application of pi networks is in the output circuits of tube-type power amplifiers, where relatively large source impedances have to be matched to relatively low antenna impedances. The configuration shown in Figure 1 forms a low-pass filter and provides substantial attenuation of harmonics of the design frequency. There is also a high-pass form of the pi network, but this configuration is not often used in Amateur Radio equipment, so I will concentrate solely on the low-pass form.

The transfer function of a pi network is defined as V_L / V_S, where V_L is the voltage across the load resistor and V_S is the source voltage. Figure 2 is a typical transfer function for a pi network, as a function of frequency. In this example, the design frequency was 1 MHz, the source and load resistances were 500 and 50 Ω, respectively, the 3 dB bandwidth was 100 kHz, and the transfer function was normalized to 0 dB at the design frequency. Notice the peak that occurs in the transfer function at the design frequency: It is the 3 dB width of this peak that defines bandwidth.

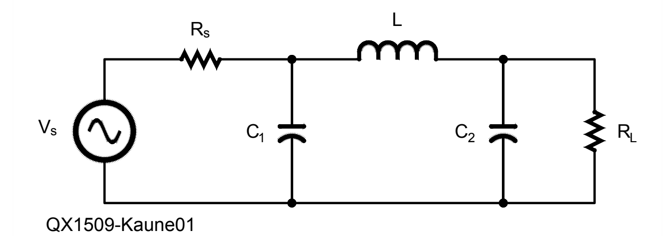

Figure 1 — This schematic diagram shows a low-pass pi network.

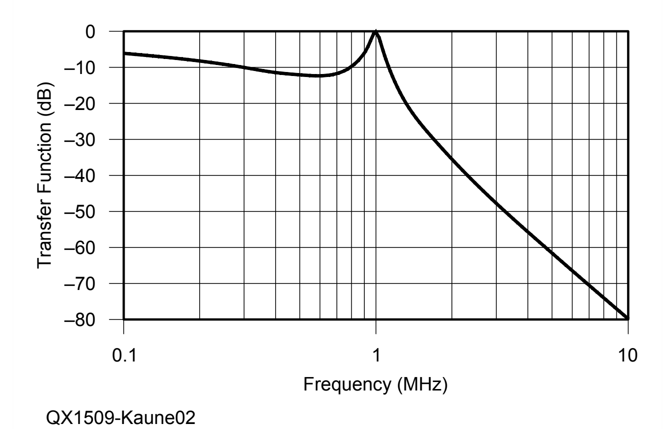

Figure 2 — Here is a plot of the transfer function (T) of a pi network. The design frequency for this example was 1 MHz. T is normalized to 0 dB at the design frequency.

I first looked into the Amateur Radio literature to see what others had learned about Q and bandwidth for pi networks. I discovered a 1983 paper by Elmer Wingfield, W5FD, titled "New and Improved Formulas for the Design of Pi and Pi-L Networks," published in *QST*.[2] In this paper, the author notes that earlier ARRL publications had defined the pi network quality factor as the expression given in Equation 3.[3]

$$Q_I = \frac{R_S}{|X_{C1}|} \quad \text{[Eq 3]}$$

where R_S and $|X_{C1}|$ are the source resistance and the magnitude of the reactance of the input capacitance. (Note that capacitive reactances are negative so, in this case, $X_{C1} = -|X_{C1}|$.) Since this definition of Q refers only to the input section of the pi network, I will refer to it as Q_I in the remainder of this paper. Wingfield argued in his paper that a better value for Q would be one that accounts for both the source and load resistances, and proposed that Q be defined by Equation 4.

$$Q_W = \frac{R_S}{|X_{C1}|} + \frac{R_L}{|X_{C2}|} \quad \text{[Eq 4]}$$

This expression, which I will refer to as Q_W, includes both input and output resistances and reactances.

The next documents I looked at were the 2003 and 2007 editions of *The ARRL Handbook for Radio Communications*, where the results reported by Wingfield were essentially reproduced.[4, 5] I then looked at the 2013 and 2015 editions of the *Handbook*, where I found a new, and slightly more complicated, definition for Q, given in Equation 5.[6, 7]

$$Q_M = \begin{cases} \dfrac{R_S}{|X_{C1}|} & \text{if } R_S > R_L \\[2ex] \dfrac{R_L}{|X_{C2}|} & \text{if } \mathrm{R_S} \le R_L \end{cases} \quad \text{[Eq 5]}$$

I will call this definition Q_M because it refers to the latest, most modern definition of Q that appears in *The ARRL Handbook*.

Finally, I am going to define a fourth expression, Q_{BW}, based on the actual bandwidth of the circuit, as calculated using Equation 2. As discussed earlier, the Amateur Radio literature uses at least three different definitions for the quality factor of a pi network. Are these definitions equivalent, and if not, which is the best predictor of the actual bandwidth of the circuit, that is, Q_{BW}?

To answer this question, I wrote a computer program (using *Visual Basic* 6.0) that, for a given pi-network design, calculates Q_I, Q_W, and Q_M, and also the transfer function from which the bandwidth and Q_{BW} can be determined. I then used this program to consider four cases, in which R_S = 5, 50, 500, and 5000 Ω, while R_L = 50 Ω. In each case, I selected values for X_{C1}, X_L, and X_{C2} so that the bandwidth of the resulting circuit was exactly 1/10 of the design frequency, that is, with $Q_{BW} = 10$. Table 1 shows my results. This table shows very clearly that the four definitions of Q are in substantial disagreement. For example, Q_I varies between 4.8 and 19.0 as the source resistance is varied from 5 to 5000 Ω, even though the bandwidth is held constant. Q_M, on the other hand, consistently indicates a smaller bandwidth than the actual value; when R_L = 5000 Ω, the indicated bandwidth is 53% of the actual value. Obviously, these two definitions of Q would be poor predictors of bandwidth (except when $R_S \approx R_L$).

The Wingfield value for Q remains consistently in the range 20.4 to 20.6 as the source resistance is varied. If we were to divide this value by 2, we would obtain values nearly equal to Q defined by bandwidth. This suggests that the definition for Q given by Equation 6 might be more useful for predicting the bandwidth of a pi network.

Table 1

Calculated *Q* Values and Harmonic Attenuations for Selected Source and Load Resistances, With Constant Bandwidth. $Q_1 = R_S / |X_{C1}|$, $Q_W = R_S / |X_{C1}| + R_L / |X_{C2}|$, $Q_M = R_S / |X_{C1}|$ if $R_S \geq R_L$ or $R_L / |X_{C2}|$ if $R_S < R_L$, and $Q_{BW} = f_0 / BW$, where f_0 = Design Frequency and BW = 3 dB Bandwidth.

Resistance (Ω)		*Reactance (Ω)*						
R_S	R_L	X_{C1}	X_L	X_{C2}	Q_1	Q_W	Q_M	Q_{BW}
5	50	1.03	4.19	3.21	4.8	20.4	15.6	10.0
50	50	4.91	9.73	4.91	10.2	20.4	10.2	10.0
500	50	32.1	41.9	10.3	15.6	20.4	15.6	10.0
5000	50	264	285	31.0	19.0	20.6	19.0	10.0

$$Q_{new} = \frac{Q_W}{2} = \frac{1}{2}\left(\frac{R_S}{|X_{C1}|} + \frac{R_L}{|X_{C2}|}\right) \quad \text{[Eq 6]}$$

To test this hypothesis, I used the computer program described earlier to determine the error estimating bandwidth using Q_{new} defined by Equation 6. I did this for a fixed load resistance of 50 Ω and source resistances of 5, 50, 500, and 5000 Ω. I found that the errors using Q_{new} to estimate bandwidth were reasonably similar across all the load resistances, so I selected the maximum error across all source resistances. The result is shown in Figure 3. Q_{new} is plotted on the horizontal axis and the vertical axis is the error using Q_{new} to estimate bandwidth. The error estimating bandwidth using Q_{new} is about –31% for Q_{new} = 3. As Q_{new} is increased, the maximum error decreases rapidly, falling to –8.6% when Q_{new} = 5 and –2.2% when Q_{new} = 10. These results show that Q_{new} is a good measure of the true bandwidth of a pi network, certainly for $Q_{new} \geq 5$. Q_{new} is a better indicator of bandwidth than any of the three Q equations that appear in *The ARRL Handbook*.

Of course, it would be desirable to also have a confirmation, based in theory, of this definition of Q [Equation 6]. In the next section, I present such a derivation.

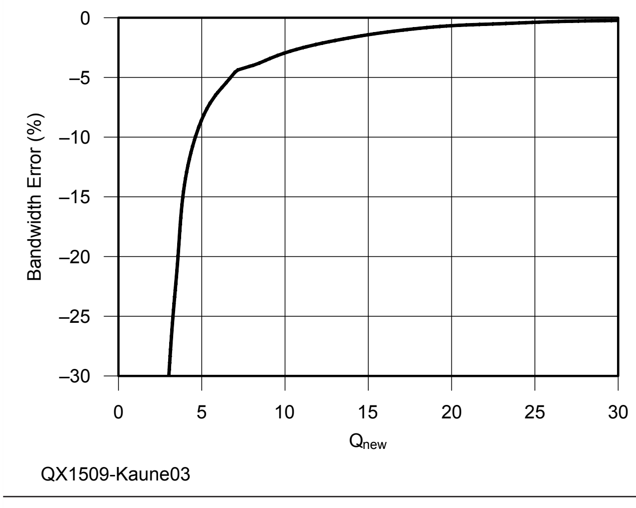

Figure 3 — This graph estimates the error in the predicted bandwidth using the quantity Q_{new}, as defined in the text.

Theoretical Derivation of Q_{new}

Various textbooks on circuit analysis show (Norton's Theorem) that a voltage source, characterized by an open-circuit voltage, V_S, and a series impedance, R_S, can be replaced by a current source, I_S, shunted by an impedance R_S, where $I_S = V_S / R_S$. Thus, the pi network drawn in Figure 4 is equivalent to the pi network in Figure 1. Next, convert the parallel combination R_S and X_{C1} in Figure 4 into a series combination R_S' and X_{C1}', and similarly convert X_{C2} and R_L into X_{C2}' and R_L', resulting in the circuit configuration shown in Figure 5. The relation between the original and converted quantities is given in Equations 7 and 8.

$$R_S' = R_S \frac{X_{C1}^2}{R_S^2 + X_{C1}^2} \text{ and } X_{C1}' = X_{C1} \frac{R_S^2}{R_S^2 + X_{C1}^2} \quad \text{[Eq 7]}$$

$$R_L' = R_L \frac{X_{C2}^2}{R_L^2 + X_{C2}^2} \text{ and } X_{C2}' = X_{C2} \frac{R_L^2}{R_L^2 + X_{C2}^2} \quad \text{[Eq 8]}$$

At first sight, Figure 5 appears to be a simple series *RLC* circuit, and, this is true so long as only one frequency is considered. Things become more complicated, however, as frequency is varied, because the resistances, R_S' and R_L', and the capacitances, C_1' and C_2' vary with frequency according to Equations 7 and 8. We can use Figure 5 as long as we restrict ourselves to the design frequency.

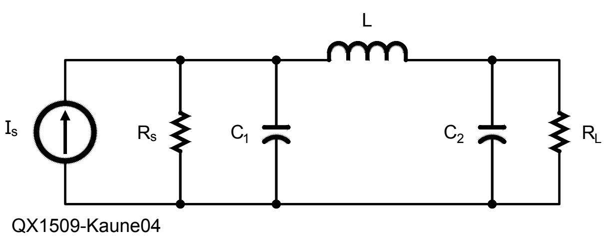

Figure 4 — This schematic diagram shows a pi network with a Norton Equivalent current source replacing the voltage source shown in Figure 1.

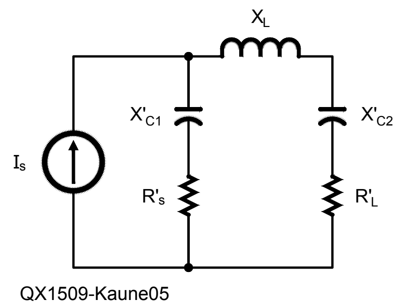

Figure 5 — Here is a redrawn version of the pi network shown in Figure 4, obtained by converting parallel resistor and capacitor combinations into equivalent series combinations.

In a matched system, the power transferred to the load resistance, R_L' in Figure 5, is maximized. It is easy to show (and is well known) that Equation 9 expresses the conditions for maximum power transfer.

$$X_L + X'_{C1} + X'_{C2} = 0$$
$$R'_S = R'_L \qquad \text{[Eq 9]}$$

These two equations determine the values of two of the three adjustable components (C_1, L, and C_2) in a pi network. We are thus free to choose the value of the remaining component. (Ultimately, we will use a specification of Q to determine the value of this component.) Thus, for the time being, assume that a value for X_{C1} has been specified. We will determine X_L and X_{C2} as functions of X_{C1}.

Use the second equation in each set shown as Equations 7, 8 and 9.

$$R'_S = R'_L \quad \Rightarrow \quad R_S \frac{X_{C1}^2}{{R_S}^2 + X_{C1}^2} = R_L \frac{X_{C2}^2}{R_L^2 + X_{C2}^2} \qquad \text{[Eq 10]}$$

Solve this equation for X_{C2}, yielding Equation 11.

$$X_{C2} = -R_L \sqrt{\frac{R_S X_{C1}^2}{R_L R_S^2 + (R_L - R_S) X_{C1}^2}} \qquad \text{[Eq 11]}$$

Then, the first equation in each set shown as Equations 7, 8 and 9 can be combined to yield Equation 12 for X_L.

$$X_L = -\left(X'_{C1} + X'_{C2}\right) = -X_{C1} \frac{R_S^2}{R_S^2 + X_{C1}^2} - X_{C2} \frac{R_L^2}{R_L^2 + X_{C2}^2} \qquad \text{[Eq 12]}$$

What is the Q of the circuit in Figure 5? Treating it as a simple series RLC circuit with total resistance $R_S' + R_L'$ and inductive reactance X_L, we get Equation 13.

$$Q = \frac{X_L}{R'_S + R'_L} \qquad \text{[Eq 13]}$$

Then, using Equations 7, 8, and 12, we get Equation 14.

$$Q = \frac{-X_{C1} \frac{R_S^2}{R_S^2 + X_{C1}^2} - X_{C2} \frac{R_L^2}{R_L^2 + X_{C2}^2}}{R_S \frac{X_{C1}^2}{{R_S}^2 + X_{C1}^2} + R_L \frac{X_{C2}^2}{R_L^2 + X_{C2}^2}} \qquad \text{[Eq 14]}$$

Equation 10 can be rearranged to yield the identity shown as Equation 15.

$$\frac{R_L^2}{R_L^2 + X_{C2}^2} = \frac{R_L}{X_{C2}^2} \frac{X_{C1}^2}{R_S} \frac{R_S^2}{R_S^2 + X_{C1}^2} \qquad \text{[Eq 15]}$$

Placing this expression into Equation 14, we obtain Equation 16.

$$Q = \frac{-X_{C1} - \frac{R_L}{R_S} \frac{X_{C1}^2}{X_{C2}}}{2 \frac{X_{C1}^2}{R_S}} = \frac{1}{2}\left(\frac{R_S}{\left|X_{C1}\right|} + \frac{R_L}{\left|X_{C2}\right|}\right) \qquad \text{[Eq 16]}$$

This is just the expression for Q that we earlier identified as Q_{new} in Equation 6. We previously found that Q_{new} is a better predictor of the bandwidth of a pi network than other definitions of Q that have been given in the Amateur Radio literature, and we have now shown that there is theoretical plausibility for this form of Q.

Calculation of Components of a Pi Network

Now I will show how to calculate X_{C1} given a value of Q. Equation 16 can be rewritten as Equation 17.

$$\left(2Q - \frac{R_S}{\left|X_{C1}\right|}\right)^2 = \frac{{R_L}^2}{X_{C2}^2} \qquad \text{[Eq 17]}$$

Use Equation 11 to eliminate X^2_{C2} in the denominator on the right-hand side of Equation 17. This gives an equation that involves only R_S, R_L, X_{C1}, and Q. Solving for X_{C1} gives the result shown in Equation 18.

$$X_{C1} = -R_S \left[\frac{2QR_S + \sqrt{4Q^2 R_S R_L - \left(R_S - R_L\right)^2}}{R_S\left(4Q^2 + 1\right) - R_L}\right] \qquad \text{[Eq 18]}$$

There is actually a second solution, obtained by replacing the plus sign in front of the square root symbol with a minus sign. I have been able to show, however, that this solution always leads to values of X_{C2} and/or X_L that have the wrong sign.

Once X_{C1} is determined, the values of X_{C2} and X_L can be calculated using Equations 11 and 12. In order for X_{C1} to be real and negative, the quantity in Equation 18 under the square root sign must not be negative and the denominator must be positive. In addition, for X_{C2}, defined by Equation 11, to be a real number, the denominator of the fraction under the square root sign must be a positive number. Analysis shows that these three conditions are met if the following inequalities are satisfied

$$Q > \frac{1}{2}\sqrt{\frac{R_S}{R_L} - 1} \quad \text{if } R_S > R_L$$
$$\text{[Eq 19]}$$
$$Q > \frac{1}{2}\sqrt{\frac{R_L}{R_S} - 1} \quad \text{if } R_S < R_L$$

Harmonic Attenuation of Pi Networks

Figure 2 shows that pi networks provide substantial attenuation of the harmonics of the design frequency. Indeed, impedance matching and harmonic attenuation are the usual reasons for employing a pi network. I used the computer program described earlier to calculate the attenuation of the second through tenth harmonics of the design frequency of the network. The results for the second, third, and fourth harmonics are shown graphically in Figure 6 for values of Q defined by Equation 6, ranging from 1 to 20, and for source impedances of 50, 500, and 5000 Ω. Because of the reciprocal nature of pi networks, harmonic attenuation for a network with a given *ratio* of source to load resistances will be the same as network whose *ratio* of source to load resistances is the inverse of the first. Thus, for example, the harmonic attenuation of a network with source and load resistances of, say, 500 and 50 Ω, respectively, will be the same as that of a network with source and load resistances of 5 and 50 Ω, respectively.

The data in Figure 6 show that harmonic attenuation increases as Q increases, but also that the rate of this increase decreases for larger values of Q. For a Q of 10, the second, third, and fourth harmonics

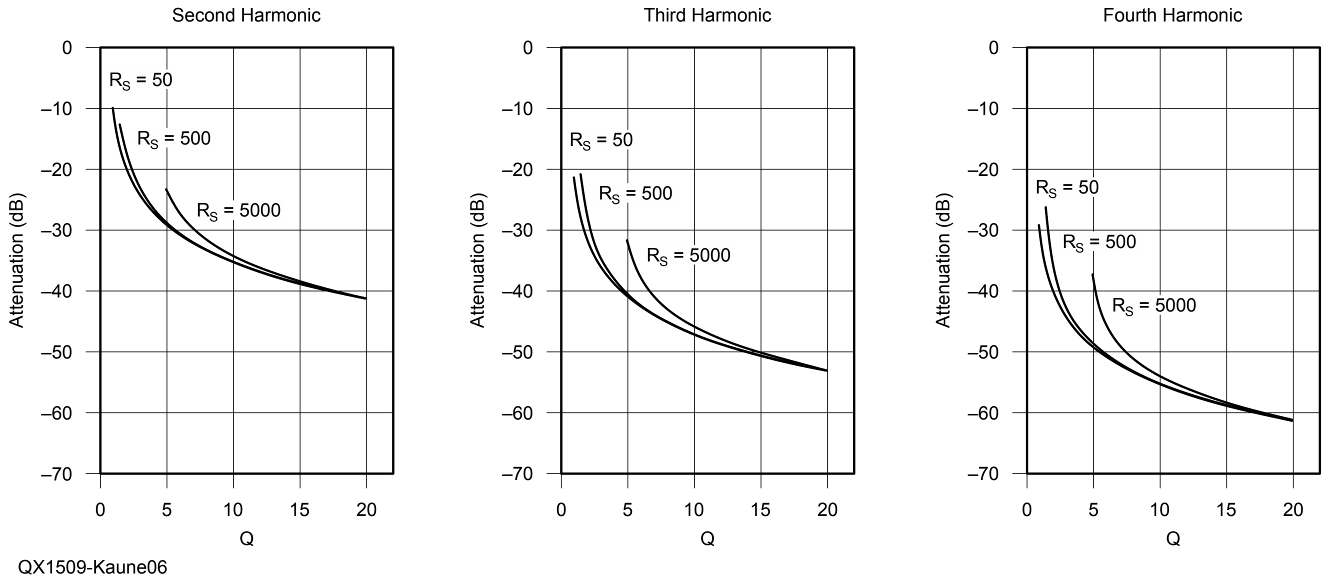

Figure 6 — These graphs plot the attenuation of second, third and fourth harmonics by a pi network as a function of *Q*. Data are shown for load resistance of 50 Ω and source resistances, R_S, of 50, 500, and 5000 Ω.

Table 2
Fitted Values for Parameters in Equation $A_N = a_0 + a_1Q + a_2Q^2$

Harmonic	*a_0 (dB)*	*a_1 (dB)*	*a_2 (dB)*	*Accuracy (dB)*
2	–22.4	–1.58	0.0316	±0.4
3	–34.0	–1.62	0.0328	±0.5
4	–41.8	–1.64	0.0333	±0.5

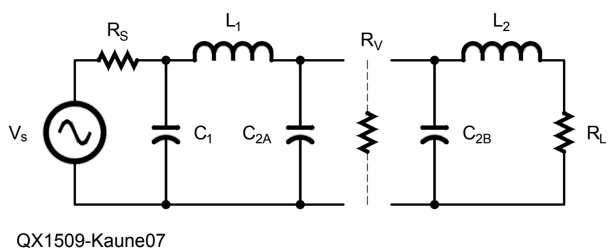

Figure 7 — Here, the pi-L network is divided into a pi network followed by an L network. The resistance looking into the input of the L network is R_V; this is also the load resistance for the pi network.

of the design frequency are attenuated by about –35 dB, –47 dB, and –54 dB, respectively, and are nearly the same for all three source resistances. The data for $Q \geq 7.5$ are described with an accuracy of ±0.5 dB or better by the form $A_N = a_0 + a_1Q + a_2Q^2$, where A_N is the attenuation of the N^{th} harmonic, and values for the constants a_0, a_1, and a_2 are listed in Table 2.

More Complex Networks

When I started this investigation, I had little idea how to *simply* analyze a pi network, and my first efforts led to quite complicated equations that were not very illuminating. It was only when I happened on the idea of transforming parallel resistances and reactances into an equivalent series pair that I was able to make progress. In this way, I was able to deduce a value for the *Q* of a pi network that effectively predicted bandwidth and harmonic attenuation, at least for larger values of *Q*. I began to wonder if the same approach would work for more complicated networks.

My first effort was to add a second inductor between the load resistor and the top of capacitor C_2 in Figure 1 to form a pi-L network. By dividing the capacitance C_2 into two parallel capacitors, C_{2A} and C_{2B}, the pi-L network can be transformed into a cascaded pair of networks, a pi network followed by a L network; this is illustrated in Figure 7. Furthermore, by selecting C_{2B} appropriately, the impedance looking into the input of the L network can be made a pure resistance, R_V; this "virtual" resistance is drawn in dotted lines in Figure 7 and is the load resistance of the pi network and the source resistance for the L network. Analysis of the L network is simple, and we can use the results earlier in this paper for the pi network.

Let Q_2 be the quality factor for the L network. By transforming the series pair L_2 and R_L into an equivalent parallel pair, the L network is transformed into a simple parallel *RLC* network, from which we can show that the reactances of L_2 and C_{2B}, and the resistance of R_V are given by the expressions of Equation 20.

$$X_{L2} = 2Q_2R_L, \quad X_{C2B} = -\left(\frac{4Q_2^2+1}{2Q_2}\right)R_L, \text{ and}$$

$$R_V = \left(4Q_2^2+1\right)R_L \qquad \text{[Eq 20]}$$

Next, let Q_1 be the quality factor for the pi network. Equations 11, 12 and 18, with L, C_2, and R_L replaced by L_1, C_{2A}, and R_V, respectively, can then be used to calculate X_{C1}, X_{C2A}, and X_{L1}. X_{C2} can then be calculated by combining X_{C2A} and X_{C2B} in parallel.

What, then, is the quality factor, *Q*, for the composite network? According to Equation 1, it is the sum of the energies stored in the pi and in the pi-L networks, divided by the power dissipated in the source and load resistors (assuming losses in C_1, C_2, L_1, and $L_2$are small enough to be neglected).

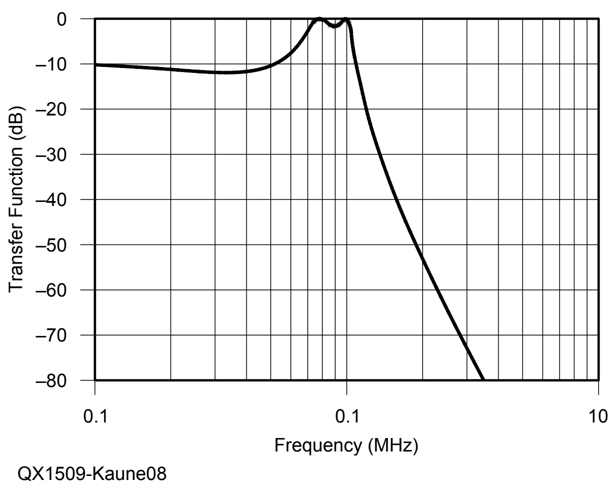

Figure 8 — This graph shows the transfer function for a cascade of two pi networks.

$$Q = 2\pi f_0 \frac{S_1 + S_2}{P_S + P_L} = 2\pi f_0 \frac{S_1}{P_S + P_L} + 2\pi f_0 \frac{S_1}{P_S + P_L} \quad \text{[Eq 20]}$$

where S_1 and S_2 are the average energies stored in the two networks, and P_S and P_L are the powers dissipated in the source and load resistances, respectively. Now look at each network separately. Network 1, the pi network, has as its load resistance R_V, which dissipates a power P_V. Then we obtain Equation 21.

$$Q_1 = 2\pi f_0 \frac{S_1}{P_S + P_V} \quad \text{[Eq 21]}$$

For network 2, the L network, we obtain Equation 22.

$$Q_2 = 2\pi f_0 \frac{S_2}{P_V + P_L} \quad \text{[Eq 22]}$$

Since R_V is matched to R_S by network 1, and R_L is matched to R_V by network 2, the powers dissipated in all three resistances are equal. With this result, we have proven Equation 23.

$$Q = Q_1 + Q_2 \quad \text{[Eq 23]}$$

I modified the computer program described earlier to model pi-L networks and found that Q calculated in this way is a good predictor of bandwidth (at least for larger values of Q). Q was not a good measure of harmonic attenuation, however, which varied significantly as the source resistance was varied.

I have explored even more complicated networks, such as two cascaded pi networks using the techniques described in this paper. With two pi networks, there are three independent parameters that have to be specified in order to uniquely define the network. For these three I chose Q_1 and Q_2, the quality factors of the individual pi networks, and R_V, the "virtual" resistance looking into the input of the second network. Figure 8 shows the transfer function for one network where $R_L = 50\ \Omega$, $R_S = 1800\ \Omega$, $R_V = 300\ \Omega$, $Q_1 = Q_2 = 5$, and with a design frequency of 1 MHz.

Note that there are now two peaks rather than one in the transfer function, leading to a considerably broadened response. In fact, the actual 3 dB bandwidth is 342 kHz. The overall Q of this network is 10, so the predicted bandwidth would be 100 kHz, substantially less than the actual value. Each pi network, taken alone, would have had a single peak at 1 MHz, but the two together yield two peaks. Evidently, there are interactions between the two pi networks as the frequency is varied away from 1 MHz. From this and other examples I have worked out, I conclude that, in general, Q is no longer related in any simple way to the network bandwidth and level of harmonic attenuation. This is consistent with the comments of Wes Hayward quoted earlier in this paper.

Discussion and Conclusions

This article has found that three existing definitions for the quality factor of pi networks found in the Amateur Radio literature are not good predictors of the bandwidths (and harmonic attenuations) of these circuits. Both empirical and theoretical analysis suggest that a better definition of Q, at least for predicting bandwidth and harmonic attenuation, is given by Equation 24.

$$Q = \frac{1}{2}\left(\frac{R_S}{|X_{C1}|} + \frac{R_L}{|X_{C2}|} \right) \quad \text{[Eq 24]}$$

where R_S and R_L are the source and load resistances, respectively, and X_{C1} and X_{C2} are the input and output capacitive reactances of the pi network, respectively.

It appears to me that the methods employed in this paper can be extended to all networks containing three reactive components, with good results.

I was able to extend my analysis to pi-L networks with good results for bandwidth prediction but not harmonic attenuation. Analysis of networks more complex than pi-L networks yielded values for Q that were not predictive of bandwidth; I am not sure that Q has much significance for these more complex networks.

There are, of course, limitations to what is presented in this paper. The two main limitations are:

1) My analysis assumes that the reactive elements are lossless, but all real inductors and, to a lesser extent, capacitors have loss.

2) The source and load resistances are assumed to be constant independent of frequency, but this is probably seldom the case. For example, pi networks are often used in power amplifiers to match the output impedance of the amplifier to an antenna, and antenna impedances vary with frequency.

It would be well in any actual design to check the performance of circuits that include pi networks using one of the modern sophisticated computer circuit modeling programs, such as *SPICE*.

During the process of the work reported here, I could not perform an exhaustive search of the technical literature on pi networks. The material in this paper is new to me, but I would not be surprised to find a paper somewhere that made similar remarks to the ones here. I hope the results are of interest to hams and other electronics experimenters.

Finally, I would like to express my appreciation to the hams that reviewed an earlier version of this paper. They identified several errors and made very helpful suggestions.

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Notes

[1]Wes Hayward, W7ZOI, Rick Campbell, KK7B, and Bob Larkin, W7PUA, *Introduction to Radio Frequency Design*, ARRL, Newington CT, 1996, p 139.

[2]Elmer A Wingfield, "New and Improved Formulas for the Design of Pi and Pi-L Networks," QST, August 1983, pp 23 – 29.

[3]ARRL, *The Radio Amateur's Handbook*, The American Radio Relay League, Inc., The Rumford Press, Concord, NH, p 51.

[4]Dana Reed, W1LC, *Ed*, *The 2003 ARRL Handbook for Radio Communications*, ARRL, 2002, Newington, CT, pp 13.6 – 13.7.

[5]Mark Wilson, K1RO, *Ed, The* 2007*ARRL Handbook for Radio Communications*, ARRL, 2006, Newington, CT, pp 18.6 – 18.7.

[6]H. Ward Silver, NØAX, *Ed*, *The* 2013*ARRL Handbook for Radio Communications*, ARRL, 2012, Newington, CT, pp 5.25 – 5.26.

[7]H. Ward Silver, NØAX, *Ed*, *The 2015 ARRL Handbook for Radio Communications*, ARRL, 2014, Newington, CT, pp 5.25 – 5.26.